PG – 207

II Semester M.Sc. Degree Examination, July 2017
(CBCS)
MATHEMATICS
M204 T : Partial Differential Equations

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　Max. Marks : 70

***Instruction** : Answer **any five** full questions. **All** questions carry **equal** marks.*

1. a) Define general and complete solutions in PDE.

   b) Eliminate c from $x^2 + y^2 + (z - c)^2 = a^2$.

   c) Find the general solution of

      i) $xu_x + yu_y = u$ and

      ii) $x^2u_x + y^2u_y = (x+y)\,u$. **(2+2+10)**

2. a) Solve $yu_x + xu_y = u$, $u(x, 0) = x^3$.

   b) Using method of characteristics solve $u_t + u^3u_x = 0, -\infty < x < \infty, t \geq 0$

      $u(x, 0) = x^{\frac{1}{3}}, -\infty < x < \infty, t \geq 0$. **(6+8)**

3. Classify the equation $\frac{\partial u^2}{\partial x^2} + x\frac{\partial u^2}{\partial y^2} = 0$ into parabolic or hyperbolic or elliptic type and reduce it to its canonical form. **14**

4. a) When is linear PDE with constant coefficients said to be reducible and irreducible ? Explain.

   b) Solve $y^2r - 2ys + t = p + 6y$ by Monge's method. **(2+12)**

5. a) Solve by appropriate Fourier transform

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2};\ 0 \leq x < \infty, t \geq 0,$$

   subject to

$$\left.\begin{array}{l} u(x, 0) = f(x) \\ \frac{\partial u}{\partial t}(x, 0) = g(x) \end{array}\right\},\ 0 \leq x < \infty,$$

$$u(0, t) = 0;\ t \geq 0.$$

P.T.O.

b) Using any method of Fourier solve

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}; 0 < x < 1; t > 0,$$

subject to

$$\left.\begin{array}{l} u(x, 0) = f(x) \\ \frac{\partial u}{\partial t}(x, 0) = g(x) \end{array}\right\}; 0 \le x \le a,$$

$$\left.\begin{array}{l} u(0, t) = 0 \\ u(a, t) = 0 \end{array}\right\}; t \in R$$

(8+6)

6. a) Solve by variable separable method the Dirichlet problem in a rectangle

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 \le x \le a; 0 \le y \le b,$$

Subject to

$$\left.\begin{array}{l} u(x, 0) = 0 \\ u(x, b) = x^2 \end{array}\right\}; 0 \le x \le a$$

$$\left.\begin{array}{l} u(0, y) = 0 \\ u(a, y) = 0 \end{array}\right\}; 0 \le y \le b$$

b) Find the solution of the three-dimensional Laplace equation in cylindrical polar coordinates. (7+7)

7. a) Make use of an appropriate Fourier transform solve the following IBVP

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}; 0 \le x < \infty, t \ge 0,$$

subject to

$$u(x, 0) = f(x); 0 \le x < \infty,$$

$$u(0, t) = 0; t \ge 0.$$

b) Solve by any method of Fourier $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$; $0 \le x \le L$; $t \ge 0$, subject to

$u(x, 0) = f(x)$; $0 \le x \le L$,

$$\left.\begin{array}{l}\frac{\partial u}{\partial x}(0,t) = 0 \\ \frac{\partial u}{\partial x}(L,t) = 0\end{array}\right\}; t \ge 0$$ (7+7)

8. Find the Green's function for the following

a) $u_t - u_{xx} = g(x)\delta(t)$; $-\infty < x < \infty$, $t > 0$ subject to $u(x, 0) = 0$; $-\infty < x < \infty$

b) $u_{tt} - c_1^2 u_{xx} = Q_1(x, t)$, $-\infty < x < \infty$, $t \ge 0$ subject to

$$\left.\begin{array}{l}u(x,0) = 0 \\ u_t(x,0) = 0\end{array}\right\}; -\infty < x < \infty$$

$$\left.\begin{array}{l}u \to 0 \\ \frac{\partial u}{\partial x} \to 0\end{array}\right\}; \text{as } |x| \to \infty$$ (7+7)